Exercises

- Compute

$$\int_0^{\pi/2} \cos x \, dx$$

$$\int_1^2 \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right) dx$$

$$\int_0^{\pi} x \sin x \, dx$$

Compute $\int_0^{\pi/2} \cos x \, dx$

- A primitive function is sin *x*, so

$$\int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} = 1 - 0 = 1$$

Compute $\int_1^2 \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right) dx$

$$\int_1^2 \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right) dx = \left[\ln x \ - 1 \times x^{-1} - \frac{1}{2} x^{-2}\right]_1^2$$

$$= \left[\ln x \ - \frac{1}{x} - \frac{1}{2x^2}\right]_1^2$$

$$= \left(\ln 2 \ - \frac{1}{2} - \frac{1}{8}\right) - \left(\ln 1 \ - 1 - \frac{1}{2}\right)$$

$$= \left(\ln 2 \ - \frac{5}{8}\right) - \left(-\frac{3}{2}\right)$$

$$= \ln 2 \ - \frac{5}{8} + \frac{12}{8} = \ln 2 \ + \frac{7}{8}$$

Compute $\int_0^{\pi} x \sin x \, dx$

- The easiest way to solve this is to directly use the formula for partial integration

$$\int_a^b u(t)v'(t)\,dt = [u(t)v(t)]_a^b - \int_a^b u'(t)v(t)\,dt$$

with u = x and v′ = sin x, so v = − cos x

$$\int_0^{\pi} x \sin x \, dx = [x \times -\cos x]_0^{\pi} - \int_0^{\pi} 1 \times -\cos x \, dx$$

$$= [-x \cos x + \sin x]_0^{\pi}$$

$$= (-\pi \times (-1) + 0) - (-0 + 0)$$

$$= \pi$$

Note

- Note that we chose $u = x$ and $v' = \sin x$. We could also have chosen $u = \sin x$ and $v' = x$
- In general, only one choice (possibly) simplifies the problem, the other choice does not lead to a simpler integral

Alternative (1/2)

- A more elegant solution is to construct a primitive. The integrand $x \sin x$ suggests something similar for a primitive, but with a $-\cos x$ in the primitive. Let's try $-x \cos x$:

  $-x \cos x$ differentiates into $-\cos x + x \sin x$
- We only need to get rid of the term $-\cos x$ .That's easy, we just need to add sin x to the primitive:

  $-x \cos x + \sin x$ differentiates into $x \sin x$

Alternative (2/2)

- So we have

$$\int_0^\pi x \sin x \, dx = [-x \cos x + \sin x]_0^\pi$$

which is identical to what we got using partial integration (2^{nd} line)